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THERMAL-PARAMETER DETERMINATION FOR THIN-WALLED DRUM-TYPE CRYSTALLIZERS

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A two-dimensional heat-transfer problem has been solved with boundary conditions of the first and third kinds for a rotating hollow thin-walled cylinder. An example of the results in use is given.

Crystallizers of drum type are used in metallurgy [1], in the chemical industry [2], and in the production of ice [3], so calculations on their thermal conditions are of interest.

Studies have been made [4-6] of the thermal fields in a rotating hollow cylinder with boundary conditions of the first kind.

In [4], a thin-walled cylinder was envisaged, with a temperature difference only around the perimeter. In [5], on the other hand, the temperature change along the cylinder director was not incorporated. In [6], the problems were solved with temperature variation along the radial and angular coordinates. The solution was presented in terms of Kelvin functions, which makes for certain difficulties in using it.

If the radius of the cylinder is greater than the wall thickness by a factor of 50 or more, the problem can be treated in Cartesian coordinates, which simplifies it considerably. This formulation may be applied to a two-dimensional plate of finite length with a conjugation condition at the ends.

In [7], the two-dimensional problem was solved for a rectangular plate with a temperature distribution on one of the surfaces varying in a specified fashion with time, while there was zero temperature at the other surfaces and a nonzero initial temperature distribution.

A real crystallizer usually works in a quasistationary state, where the initial temperature distribution is unimportant and the ends of the plate have identical nonzero but unknown temperature distributions over the thickness, while Newton's law applies to the heat transfer at the cooled surface. The problem is formulated mathematically as

$$\frac{1}{a}\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2},\quad \begin{matrix}0\leqslant x\leqslant\delta,\\ 0\leqslant y\leqslant l=2\pi R,\end{matrix} \tag{1}$$

$$\theta(x,\ y,\ 0)=0,\quad \theta(x,\ 0,\ \tau)=\theta(x,\ l,\ \tau), \tag{2}$$

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$$\frac{\partial\theta(0,\ y,\ \tau)}{\partial x}=\frac{\mathrm{Bi}}{\delta}\ \theta(0,\ y,\ \tau), \tag{3}$$

$$\begin{cases}\theta(\delta,\ y,\ \tau)=\dfrac{C_0}{2}+\displaystyle\sum_{m=1}^{\infty}\left[C_m\cos\left(\omega\tau+\frac{my}{R}\right)+D_m\sin\left(\omega\tau+\frac{my}{R}\right)\right],\\ \omega=\dfrac{2\pi m}{\tau_c}.\end{cases} \tag{4}$$

As we lack a boundary condition along the y coordinate, from (4) we get two equations

$$\theta(\delta,\ 0,\ \tau)=\frac{C_0}{2}+\sum_{m=1}^{\infty}(C_m\cos\omega\tau+D_m\sin\omega\tau), \tag{5a}$$

$$\theta\left(\delta,\ \frac{l}{2},\ \tau\right)=\frac{C_0}{2}+\sum_{m=1}^{\infty}[C_m\cos(\omega\tau+\pi m)+D_m\sin(\omega\tau+\pi m)]. \tag{5b}$$

We apply a Laplace transformation with respect to time to (1) and use (2) to get

$$\frac{s}{a}\ L=\frac{\partial^2 L}{\partial x^2}+\frac{\partial^2 L}{\partial y^2}. \tag{6}$$

The integral of (6) takes the form

$$L(x,\ y,\ s)=N_1\operatorname{ch}kx+N_2\operatorname{sh}kx+N_3\operatorname{ch}ky+N_4\operatorname{sh}ky;\quad k=\sqrt{\frac{s}{a}}. \tag{7}$$

The constants of integration are found from boundary conditions (2), (3), and (5):

$$L=\left[\frac{C_0}{2s}+\sum_{m=1}^{\infty}\frac{C_m s+D_m\omega}{s^2+\omega^2}\left(1-\frac{\varphi\operatorname{sh}kl}{\operatorname{sh}kl-2\operatorname{sh}\frac{kl}{2}}\right)\right]\times$$

$$\times\frac{k\delta\operatorname{ch}kx+\mathrm{Bi}\operatorname{sh}kx}{k\delta\operatorname{ch}k\delta+\mathrm{Bi}\operatorname{sh}k\delta}+\frac{\operatorname{sh}k(l-y)+\operatorname{sh}ky}{\operatorname{sh}kl-2\operatorname{sh}\frac{kl}{2}}\ \frac{k\delta+\mathrm{Bi}\operatorname{sh}kx}{k\delta+\mathrm{Bi}\operatorname{sh}k\delta}\ \sum_{m=1}^{\infty}\varphi\ \frac{C_m s+D_m\omega}{s^2+\omega^2},\quad \varphi=1-\cos\pi m. \tag{8}$$

We use the expansion theorem for simple and multiple roots to return to the original:

$$\theta(x,\ y,\ \tau)=\frac{C_0}{2}\left[\frac{1+\mathrm{Bi}\,z}{1+\mathrm{Bi}}+\sum_{m=1}^{\infty}\frac{F(z,\ \tau)}{\mu_n^2}\right]+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{D_m\mathrm{Pd}-C_m\mu_n^2}{\mathrm{Pd}^2+\mu_n^4}\left(1-\frac{\sin\mu_n\frac{l}{\delta}}{\sin\mu_n\frac{l}{\delta}-2\sin\mu_n\frac{l}{2\delta}}\right)F(z,\ \tau)$$

$$+\sum_{m=1}^{\infty}[(M_i+M_{-i})\cos\omega\tau+i(M_i+M_{-i}\sin\omega\tau)]; \tag{9}$$

$$F(z,\ \tau)=A_n\left(\cos\mu_n z+\frac{\mathrm{Bi}}{\mu_n}\ \sin\mu_n z\right)\exp(-\mu_n^2\mathrm{Fo}),$$

$$A_n=\frac{2\mu_n^2\sqrt{\mu_n^2+\mathrm{Bi}^2}}{\mathrm{Bi}^2+\mathrm{Bi}+\mu_n^2},\quad \mu_n+\mathrm{Bi}\operatorname{th}\mu_n=0,$$

$$M_{\pm i}=\frac{C_m\mp iD_m}{2}\left[\varphi\ \frac{p+\mathrm{Bi}\operatorname{sh}pz}{p+\mathrm{Bi}\operatorname{sh}p}\ \frac{\operatorname{sh}q(1-\gamma)+\operatorname{sh}q\gamma}{\operatorname{sh}q-2\operatorname{sh}\frac{q}{2}}+\frac{p\operatorname{ch}pz+\mathrm{Bi}\operatorname{sh}pz}{p\operatorname{ch}p+\mathrm{Bi}\operatorname{sh}p}\left(1-\frac{\varphi\operatorname{sh}q}{\operatorname{sh}q-2\operatorname{sh}\frac{q}{2}}\right)\right],$$

$$p=\sqrt{\pm i\mathrm{Pd}},\quad q=\frac{pl}{\delta},\quad z=\frac{x}{\delta},\quad \gamma=\frac{y}{l}.$$

The temperature distribution over the wall thickness in the rotating cylinder is the point of interest, and for any arbitrarily taken point y_j it differs only in the time origin, which is of no interest, so we take $y_j = 0$ as fixed point and

$$\theta(x,\ 0,\ \tau)=\theta_d(x,\ \tau). \tag{10}$$

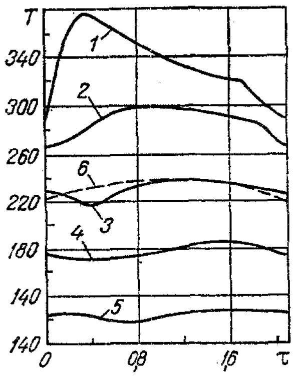

Fig. 1

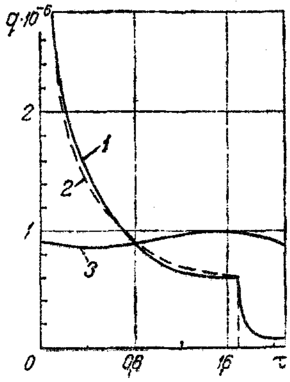

Fig. 2

Fig. 1. Time dependence of the crystallizer wall temperature: 1-5) correspondingly for $z = 1$, 0.75, 0.5, 0.25, and 0; 6) temperature averaged over the wall thickness; T in °C and τ in sec.

Fig. 2. Dependence of the heat flux on time as calculated from (14): 1 and 3) for $z = 1$ and 0 as calculated from the Stefan solution [7]; 2) for $z = 1$; $q \cdot 10^6$, W/m².

The definition of wall thinness $l/\delta > 100\pi$ means that $\delta^2/a > 0.01$ for all possible crystallizers. Then with these values

$$q > 30, \quad \frac{\operatorname{sh} q}{\operatorname{sh} q - 2\operatorname{sh}\dfrac{q}{2}} = 1. \tag{11}$$

From (9) with (11) we get as follows for the quasistationary heat-transfer state (Fo > 6):

$$\theta_d(x,\ \tau) = \frac{C_0}{2}\,\frac{1+\mathrm{Bi}\,z}{1+\mathrm{Bi}} + \sum_{m=1}^{\infty} \frac{\operatorname{ch}\alpha}{\operatorname{ch}\beta}\,[(1-\varphi)(u_1v_1+u_2v_2)+\varphi(u_3v_3+u_4v_4)], \tag{12}$$

$$\theta_d(\tau) = \frac{1}{\delta}\int_0^{\delta}\theta_d dx = \frac{C_0}{2}\cdot\frac{1+0.5\mathrm{Bi}}{1+\mathrm{Bi}} + \sum_{m=1}^{\infty}\frac{1}{2\beta}\,[(1-\varphi)(u_1^0 v_1+u_2^0 v_2)+\varphi(u_3^0 v_3+u_4^0 v_4)], \tag{13}$$

$$q(z,\ \tau) = \lambda\,\frac{\partial\theta_d(x,\ \tau)}{\partial x} = \frac{\lambda}{\delta}\left\{\frac{C_0}{2}\,\frac{\mathrm{Bi}}{1+\mathrm{Bi}} + \sum_{m=1}^{\infty}\beta\,\frac{\operatorname{ch}\alpha}{\operatorname{ch}\beta}\,[(1-\varphi)\quad(u_1' v_1+u_2' v_2)+\varphi(u_3' v_3+u_4' v_4)]\right\}, \tag{14}$$

$$u_1 = \operatorname{th}\alpha(\sigma\cos\alpha-\sin\alpha)+\cos\alpha,\quad u_2 = \sin\alpha(\sigma+\operatorname{th}\alpha)+\cos\alpha,$$

$$u_3 = \sigma\operatorname{th}\alpha\cos\alpha+\operatorname{sch}\alpha,\quad u_4 = \sigma\sin\alpha+\operatorname{sch}\alpha,$$

$$u_1^0 = \sigma(\operatorname{th}\beta\sin\beta+\cos\beta-\operatorname{sch}\beta)+2\operatorname{th}\beta,$$

$$u_2^0 = \sigma(\operatorname{th}\beta\sin\beta-\cos\beta+\operatorname{sch}\beta)-2\sin\beta,$$

$$u_3^0 = \sigma(\operatorname{th}\beta\sin\beta+\cos\beta-\operatorname{sch}\beta)+2\beta\operatorname{sch}\beta,$$

$$u_4^0 = \sigma(\operatorname{th}\beta\sin\beta-\cos\beta+\operatorname{sch}\beta)-2\beta\operatorname{sch}\beta,$$

$$u_1' = \sigma(\cos\alpha-\operatorname{th}\alpha\sin\alpha)-2\sin\alpha,\quad u_2' = \sigma(\cos\alpha+\operatorname{th}\alpha\sin\alpha)+2\operatorname{th}\alpha\cos\alpha,$$

$$u_3' = \sigma(\cos\alpha-\operatorname{th}\alpha\sin\alpha),\quad u_4' = \sigma(\cos\alpha+\operatorname{th}\alpha\sin\alpha),$$

$$v_1 = \frac{(C_mG_1-D_mG_2)\cos\omega\tau+(C_mG_2+D_mG_1)\sin\omega\tau}{G_1^2+G_2^2},$$

$$v_2 = \frac{(C_mG_2+D_mG_1)\cos\omega\tau-(C_mG_1-D_mG_2)\sin\omega\tau}{G_1^2+G_2^2},$$

$$v_3 = \frac{(C_m G_3 - D_m G_4)\cos\omega\tau + (G_m G_4 + D_m G_3)\sin\omega\tau}{G_3^2 + G_4^2},$$

$$v_4 = \frac{(C_m G_4 + D_m G_3)\cos\omega\tau - (C_m G_3 - D_m G_4)\sin\omega\tau}{G_3^2 + G_4^2},$$

$$G_1 = \text{th}\,\beta\,(\sigma\cos\beta - \sin\beta) + \cos\beta, \quad G_2 = \sin\beta\,(\sigma + \text{th}\,\beta) + \cos\beta,$$

$$G_3 = \sigma\,\text{th}\,\beta\cos\beta + \text{sch}\,\beta, \quad G_4 = \sigma\sin\beta + \text{sch}\,\beta,$$

$$\beta = \sqrt{\frac{\text{Pd}}{2}}, \quad \alpha = \beta z, \quad \sigma = \frac{\text{Bi}}{\beta}.$$

The condition for transferring to the one-dimensional case in (12)-(14) is $\varphi = 0$.

As a rule, the temperature gradient in the cross section of a crystallizer substantially exceeds the gradient along the y coordinate, so we test the desirability of using the formulas for the two-dimensional case.

We consider a steel crystallizer of drum type with diameter $D_{av} = 1.01$ and wall thickness $\delta = 0.01$ in an experimental equipment for freezing steel-casting slags, which is cooled from within by boiling water. The temperature of the outer surface was measured with a pocketed thermocouple. The resulting curve for the steady-state period of heat transfer was averaged, approximated, and expanded as a Fourier series. The heat flux at the outer surface was determined by solving the Stefan's problem with a known wall temperature. The Biot number was calculated with known mean values for the temperature and heat flux. From these data we calculated the temperature distribution and the heat flux for the largest and smallest crystallizer speeds allowed by the technological parameters.

Figures 1 and 2 show the results for the maximum speed of 28.6 rpm. The heat-flux curve for the hot surface deviates from the curve calculated by solving the Stefan problem evidently because of the inaccuracy in the averaging or in approximating the experimental data.

The calculations were performed for the two-dimensional and one-dimensional ($\varphi = 0$) cases. The discrepancies in the temperatures did not exceed 1.4% for the maximum speed or 2.3% for the minimum one.

The heat fluxes differed considerably:

$$\Delta q(z, \tau) = \frac{q_d(z, \tau) - [q(z, \tau)]_{\varphi=0}}{[q(z, \tau)]_{\varphi=0}} \cdot 100\%,$$

$$-7 \leqslant \Delta q_{max}(1, \tau) \leqslant 6.56; \quad -2.8 \leqslant \Delta q_{max}(0, \tau) \leqslant 3.1,$$

$$-21.6 \leqslant \Delta q_{min}(1, \tau) \leqslant 13.8; \quad -6.9 \leqslant \Delta q_{min}(0, \tau) \leqslant 8.8.$$

With $\text{Bi} = \infty$, we obtained a solution from (12) for boundary conditions of the first kind and calculated the temperature pattern under identical conditions from (12) and from the solution in the rigorous formulation [6] for y = 0. The discrepancies were not more than 0.8% for the minimum speed.

Therefore, the solution to (12) gives satisfactory accuracy with more general boundary conditions at the cooled surface and is also considerably more convenient to use than the solutions to analogous problems in [4-6].

One can neglect the temperature gradient along the y coordinate in order to simplify the calculation further only when the thermal activity of the crystallizer material exceeds that of the crystallizing material by an order of magnitude or more and is determined only by the temperature pattern in the crystallizer.

NOTATION

a, λ, thermal diffusivity and thermal conductivity, respectively; x, y, τ, coordinates and time τ; δ, l, R, thickness length, and radius of the crystallizer wall; θ, temperature; q, heat flux; ω, angular frequency of the m-th harmonic; C_0, C_m, D_m, Fourier expansion coefficients; L, s, Laplace transforms of temperature and time; Bi, Pd, Biot and Predvoditelev numbers, respectively; $i = \sqrt{-1}$; n, m = 1, 2, 3, ..., ∞; subscripts d, two-dimensional problem; av, mean value; c, thermal effect cycle.

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